An Injection Island GA for Flywheel Design Optimization

David Eby, R. C. Averill
Department of Materials Science and Mechanics
Boris Gelfand, William F. Punch, III, Owen Mathews, Erik D. Goodman
Genetic Algorithms Research and Applications Group (GARAGe)
Michigan State University, East Lansing, MI 48824 USA
Phone (517)355-6453 Fax (517)432-0704
goodman@egr.msu.edu

ABSTRACT

This paper presents an approach to optimal design of elastic flywheels using an Injection Island Genetic Algorithm (iiGA). An iiGA in combination with a finite element code is used to search for shape variations to optimize the Specific Energy Density (SED) of elastic flywheels. SED is defined as the amount of rotational energy stored per unit mass. iiGAs seek solutions simultaneously at different levels of refinement of the problem representation (and correspondingly different definitions of the fitness function) in separate sub-populations (islands). Solutions are sought first at low levels of refinement with an axisymmetric plane stress finite element code for high speed exploration of the coarse design space. Next, individuals are injected into populations with a higher level of resolution that uses an axisymmetric three dimensional finite element model to "fine-tune" the flywheel designs. In true multi-objective optimization, various "sub-fitness" functions can be defined that represent "good" aspects of the overall fitness function. Solutions can be sought for these various "sub-fitness" functions on different nodes and injected into a node that evaluates the overall fitness. Allowing subpopulations to explore different regions of the fitness space simultaneously allows relatively robust and efficient exploration in problems for which fitness evaluations are costly.

1.0 INTRODUCTION

This paper will describe the advantages of searching with an axisymmetric plane stress finite element model (with a "sub-fitness" function) to quickly find building blocks needed to inject into an axisymmetric three-dimensional finite element model through use of an iiGA. An optimal annular composite flywheel shape will be sought by an iiGA and, for comparison, by a "ring" topology parallel GA. The flywheel is modeled as a series of concentric rings (see Figure 1). The thickness of each ring varies linearly in the radial direction with the possibility for a diverse set of material choices for each ring. Figure 2 shows a typical flywheel model in which symmetry is used to increase computational efficiency. The overall fitness function for the genetic algorithm GALOPPS was the specific energy density (SED) of a flywheel, which is defined as:

$$SED = \frac{\frac{1}{2} I\omega^2}{mass} \qquad 1.)$$

where ω is the angular velocity of the flywheel ("sub-fitness" function), I is the mass moment of inertia defined by:

$$I = \int_V \rho \cdot r^2 dV \qquad 2.)$$

and ρ is the density of the material.

Recently iiGAs have been successfully applied in the optimization of laminated composite materials [1,2]. An eventual goal of this effort is to develop tools for multi-disciplinary optimization of large scale 3-dimensional composite structures. Combining a GA with the finite element method is by now a familiar approach in the optimization of structures, but using an iiGA with multiple evaluation tools and with different fitness functions is a new approach aimed at decreasing computational time while increasing the robustness of a typical GA.

1.1 PARALLEL GENETIC ALGORITHMS

Two problems associated with GAs are their need for many fitness evaluations and their propensity to converge prematurely. An approach that ameliorates both of these problems is a parallel GA (PGA), which also produces a more realistic model of nature than a single large population. PGAs both decrease processing time and better explore the search space.

Unlike some specialized sequential GAs which pay a high computational cost for maintaining subpopulations based on similarity comparisons (niching techniques, etc.), PGAs maintain multiple, separate subpopulations which may be allowed to evolve nearly independently. This allows each subpopulation to explore different parts of the search space, each maintaining its own high-fitness individuals and each controlling how mixing occurs with other subpopulations, if at all.

1.2 ISLAND INJECTION GAs

iiGAs represent an approach to search at various levels of resolution within a given space. This includes first searching at low levels of resolution on different nodes (islands) and then injecting the high-performance individuals into an island of higher resolution to “fine-tune” them. Figure 2a shows the iiGA used to perform multiple refinements in the geometric representation by increasing the number of rings in flywheels for this paper. For composite analysis, material properties can vary from ring to ring. Islands with different levels of resolution evaluate fitness with a either simplified analysis that is computationally cheaper or a refined computationally expensive analysis. Different GA parameters can be used for each population. The rate of crossover, mutation, and island interaction can all vary from island to island. For example, islands can exploit a simplified evaluation tool that is computationally cheap by increasing the local island population size. Also, islands with a computationally cheap evaluation tool can be expedited further by increasing the number of generations evaluated before injecting the information to other islands.

Multi-objective optimization requires combining many parameters to find an optimal design. Each individual fitness measure may have its own optimal or sub-optimal solutions. By using iiGAs, each individual parameter could be used as a “sub-fitness” function since it represents “good” designs within the search space. iiGAs take advantage of the low communication required to migrate individuals from island to island. Often, only the best individual in a population is migrated to allow “good” ideas (building blocks) to be combined with other “good” ideas to find “better” ideas amongst islands of different “sub-fitness” functions. Next, the individuals are injected into a final node where the evaluation of an overall fitness function is employed. This search method ensures a robust exploration of the search space for all aspects of the overall fitness. Of course, many variations on these island injection architectures can be custom tailored for specific problems.

iiGAs have the following advantages over other PGAs.

(i) Building blocks of lower resolution can be directly found by search at that resolution. After receiving lower resolution solutions from its parent node(s), a node of higher resolution can ``fine-tune'' these solutions.

(ii) The search space in nodes with lower resolution is proportionally smaller. This typically results in finding ``fit'' solutions more quickly, which are injected into higher resolution nodes for refinement.

(iii) Nodes connected in the hierarchy (nodes with a parent-child relationship) share portions of the same search space, since the search space of parent is contained in the search space of child. Fast search at low resolution by the parent can potentially help the child find fitter individuals.

(iv) iiGAs embody a divide-and-conquer and partitioning strategy which has been successfully applied to many problems. Homogeneous PGAs cannot guarantee such a division since crossover and mutation may produce individuals that belong to many subspaces, i.e., the divisions cannot be maintained. In iiGAs, the search space is fundamentally divided into hierarchical levels with well defined overlap (the search space of the parent is contained in the search space of the child).

(v) In iiGAs, nodes with smaller block size can find the solutions with higher resolution. Although Dynamic Parameter Encoding (DPE) and ARGOT also deal with the resolution problem, using a zoom or inverse zoom operator, they are different from iiGAs. First, they are working at the phenotype level and only for real-valued parameters. iiGAs divide the string into small blocks regardless of the meaning of each bit. Second, it is difficult to establish a well-founded, general trigger criterion for zoom or inverse zoom operators in PDE and ARGOT. Furthermore, the sampling error can fool them into prematurely converging on suboptimal regions. Unlike PDE and ARGOT, iiGAs search different resolution levels in parallel and eliminate the risk of zooming into the wrong target interval, although there remains some risk that search will prematurely converge on a suboptimal region.

1.3 PARALLEL COMPUTATIONAL ENVIRONMENT

The parallel environment in which we first implemented these ideas was based on a modification to GAucsd established by using a modified P4 on a network of Sparc 10 workstations, and has now been more flexibly realized in the GALOPPS toolkit developed by the authors. The software can be run on one or multiple workstations [3].

2.0 FINITE ELEMENT MODELS OF FLYWHEELS

Two axisymmetric finite element models were developed to predict planar and three-dimensional stresses that occur in flywheels composed of orthotropic materials undergoing a constant angular velocity. Both finite element models were developed applying the principle of minimum potential energy. The finite element model that assumes a plane stress state is truly a one-dimensional finite element model, and is accurate when the gradient of the ring thickness is small. The finite element model that yields a three-dimensional stress state is truly a two-dimensional finite element model, and is accurate for all shapes. An automated mesh generator was written to allow for element refinement through the transverse normal and the radial directions. Therefore, the finite element code that predicts three-dimensional stresses can have various levels of refinement. A coarse mesh with a small number of degrees of freedom will be less accurate but more efficient than a refined mesh that contains more degrees of freedom. The mesh was also generated to minimize the time required to solve the set of linear equations created by the finite element code. By first assuming an initial angular velocity, the stresses and strains were calculated. Next, the initial angular velocity was scaled to the maximum failure angular velocity. The maximum stress failure criterion was used to predict the maximum failure angular velocity in the analysis of isotropic flywheels, while the maximum strain criterion was used for composite flywheels.

3.0 RESULTS AND DISCUSSION

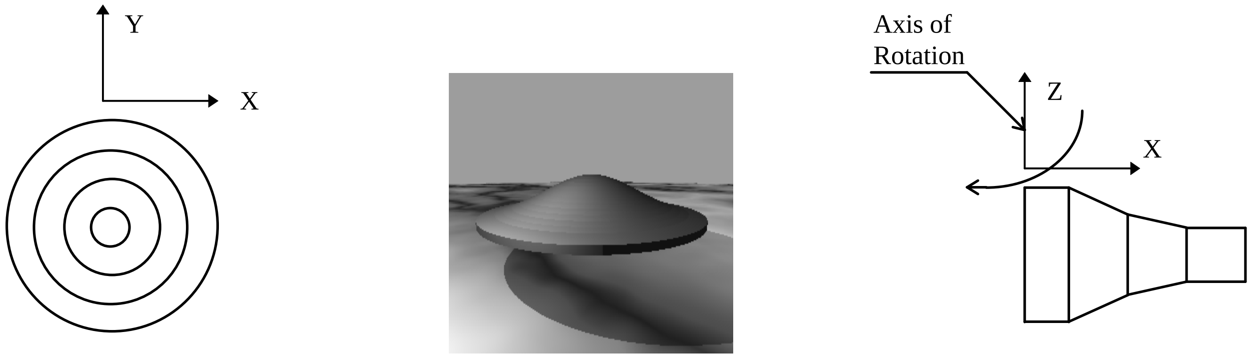
The search space for the plane stress finite element model evaluation contains good building blocks for the iiGA. Also, the plane stress evaluation (0.001seconds per evaluation) is up to a 1000 times faster than the three-dimensional evaluation of stress (for this analysis). To make the GA search less computationally intensive and more robust, an iiGA as shown in Figure 2a was designed. Islands 0 through 2 evaluate fitness based on angular velocity with a simplified plane stress finite element model with varying geometric resolutions (3, 6 and 12 rings). Islands 0 through 2 have 7, 13 and 25 degrees of freedom respectively. Islands 3 through 11 measure fitness based on SED using the three-dimensional axisymmetric finite element model. Islands 3 and 4 are low in geometric resolution (3 rings), but have 160 degrees of freedom. Islands 5 and 6 are medium in geometric resolution (6 rings), containing 558 degrees of freedom. Islands 7 and 8 are high in geometric resolution (12 rings), having 1952 degrees of freedom. Islands 9 through 11 are the highest in geometric resolution (24 rings) with 9,982 degrees of freedom.

A full cycle consists of evaluating a specified number of generations (which varies from island to island) in the island injection topology. Typically, for a larger, computationally expensive problem, each island would be located on a separate processor. For this problem, only a single Sun Sparc Ultra workstation was used. Islands 0 through 2 had a 75% rate of crossover, population size of 300, and completed 15 generations per cycle before migrating the island's best individual in accordance with Figure 2a. Islands 3 and 4 had a crossover rate of 70%, population size of 100, and completed 7 generations per cycle before migrating 3 individuals. Islands 5 and 6 had a 65% crossover rate, population size of 130 and completed 4 generations before migrating individuals. Islands 7 and 8 had a crossover rate of 65%, population size of 130 and migrated individuals after evaluating 3 generations. Islands 9 through 11 had a crossover rate of 60%, population size of 100 and received migrated individuals every 3 generations. Islands 0 through 2 will converge much faster to "good" building blocks when compared to the rest of the islands due to the simplification of the plane stress evaluation and the level of resolution. The iiGA topology design in Figure 2a uses this as an advantage because the topology injects building blocks from the simplified plane stress evaluation based on angular velocity into two isolated islands that evolve independently, searching separate spaces efficiently using the axisymmetric three-dimensional finite element model to evaluate SED. A "ring" PGA topology, shown in Figure 3a, contains 20 islands with the same total number of individuals (divided equally among the islands) as the iiGA topology shown in Figure 2a. The flywheel geometric resolution was 24 rings, containing 9,982 degrees of freedom.

The composite flywheel material was E-glass, a quasi-isotropic material. Figure 2b displays the "best ever" annular composite flywheel at all the levels of geometric resolution. Also, Figure 2b compares the three-dimensional to the plane stress axisymmetric results. The plane stress results based on angular velocity are exaggerated shapes that are artifacts of the analysis. The plane stress results cannot be fully dismissed because they are the building blocks that helped form the final "finely tuned" flywheels. Figure 3b displays the best ever flywheel found in the "ring" topology, which ran for the same amount of time (3 hours). Clearly the iiGA is more efficient in its search technique than the "ring" PGA. Also, the iiGA annular composite flywheel has a 35% increase in SED.

4.0 CONCLUSIONS

The iiGA can approach multi-disciplinary optimization problem in a unique way. For many problems, the iiGA can be used to break down a complex a fitness function into "sub-fitness" functions, which represent "good" aspects of the overall fitness. Also, the iiGA can use differing evaluation tools. A simplified analysis tool can be used to quickly search for good building blocks. This, in combination with searching at various levels of resolution, makes the iiGA efficient and robust. Mimicking a smart engineer, the iiGA can first quickly evaluate the overall response of a structure with a coarse representation of the design and finish the job off by slowly increasing the levels of refinement until a "finely tuned" structure has been evolved. This approach allows the iiGA to decrease computational time and increase the robustness of a typical GA. This was demonstrated with the comparison of the optimized annular composite flywheel from the "ring" type GA to the iiGA, where a 35% increase in SED was found by the iiGA.



Top View 3-D View Side View

Figure1. Visual Display of Flywheel

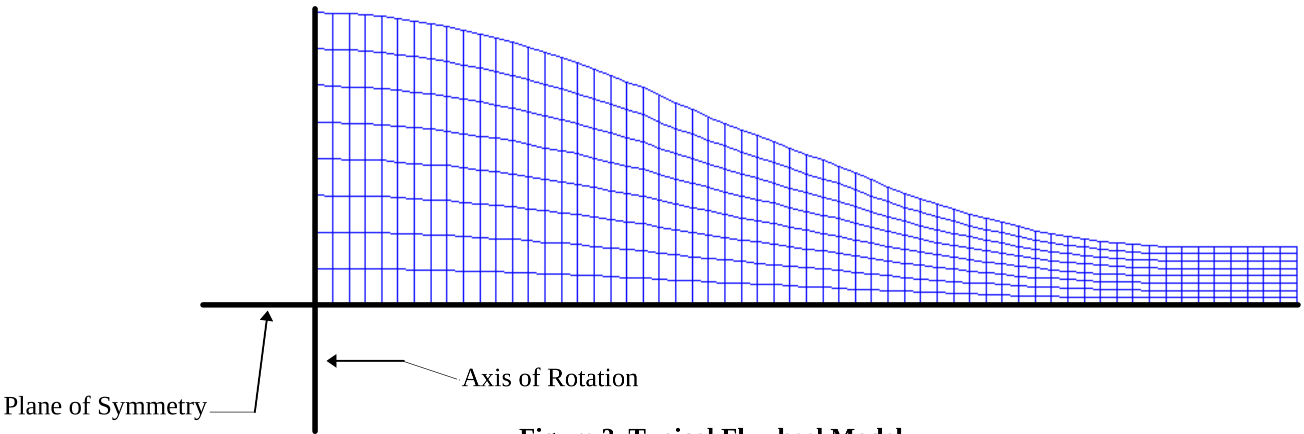
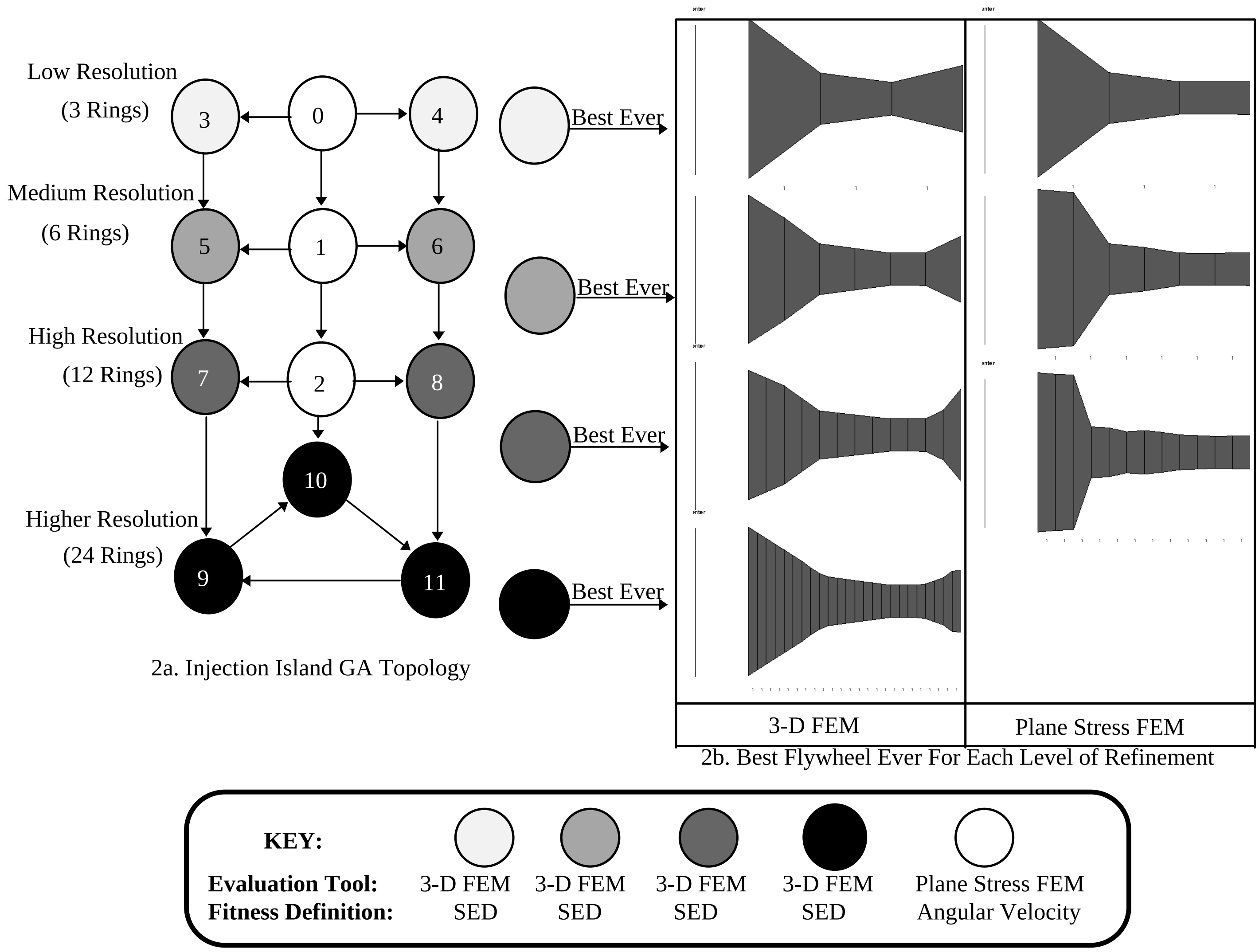


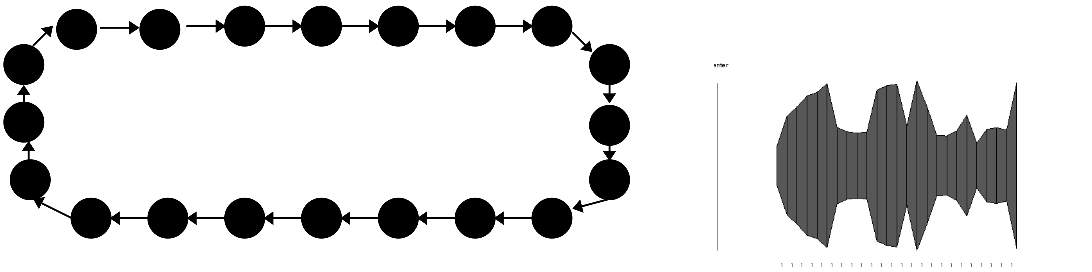
Figure 2. Typical Flywheel Model



2a. Injection Island GA Topology

2b. Best Flywheel Ever For Each Level of Refinement

Figure 2. Injection Island GA Topology with Annular Composite Flywheel Results.



3a. "Ring" PGA Topology.

3b. Best Ever Flywheel from "Ring" PGA within the time used by iiGA.

Figure 3. 20-Ring Island GA Topology with Annular Composite Flywheel Results.

REFERENCES

1. W. F. Punch III, R.C. Averill, E.D. Goodman, S. C. Lin, and Y. Ding, February 1995,"Design Using Genetic Algorithms - Some Results for Laminated Composite Structures", *IEEE Expert*, Vol. 10 (1), pp. 42-49.
2. W. F. Punch III, R.C. Averill, E.D. Goodman, S. C. Lin, Y. Ding, and Y.C. Yip, 1994, "Optimal Design of Laminated Composite Structures using Coarse-Grain Parallel Genetic Algorithms," *Computing Systems in Engineering*, Vol. 5, No. 4-6, pp. 414-423.
3. E. D. Goodman, R. C. Averill, W. F. Punch III and D. J. Eby, 1997, "Parallel Genetic Algorithms in the Optimization of Composite Structures", WSC2 Proceedings , World Wide Web Conference, www.egr.msu.edu/~ebydavid/ver5.html